Homework 9

1. Evaluate the limit, using L'Hopital's Rule if necessary. .

$$\lim_{x \to \infty} (1+x)^{\frac{1}{x}}$$

2. Find the derivative of the function.

$$f(x) = arc\sec 2x$$

3. Find or evaluate the intergral.

(a)

$$\int \frac{1}{\sqrt{-2x^2+8x+4}} \mathrm{d}x$$

(b)

$$\int_1^3 \frac{1}{\sqrt{x}(1+x)} \mathrm{d}x$$

Sol：

1.

$$\text{Let } y = \lim_{x\to\infty}(1+x)^{\frac{1}{x}}$$

$$\ln y = \lim_{x\to\infty}\frac{\ln(1+x)}{x} = \lim_{x\to\infty}\frac{\left(\frac{1}{1+x}\right)}{1} = 0$$

$$\text{So , } \ln y = 0, y = e^0 = 1$$

$$\text{Therefore, } \lim_{x\to\infty}(1+x)^{\frac{1}{x}} = 1$$

2.

$$f(x) = arc\sec 2x$$

$$f'(x) = \frac{2}{|2x|\sqrt{4x^2-1}} = \frac{1}{|x|\sqrt{4x^2-1}}$$

3.(a)

$$\int \frac{1}{\sqrt{-2x^2+8x+4}}\,\mathrm{dx} = \int \frac{1}{\sqrt{12-2(x^2-4x+4)}}\,\mathrm{d}x$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{6-(x-2)^2}}\,\mathrm{d}x$$

$$= \frac{1}{\sqrt{2}}\,\mathrm{arc\,sin}\left(\frac{x-2}{\sqrt{6}}\right) + \mathrm{C}$$

$$= \frac{\sqrt{2}}{2}\arcsin\left[\frac{\sqrt{6}}{6}(x-2)\right] + \mathrm{C}$$

(b)

$$\text{Let } \mathrm{u} = \sqrt{x}, u^2 = x, 2\mathrm{u}\,\mathrm{du} = \mathrm{d}x, 1 + x = 1 + \mathrm{u}^2$$

$$\int_1^{\sqrt{3}} \frac{2\mathrm{u}}{\mathrm{u}(1+\mathrm{u}^2)}\,\mathrm{du} = \int_1^{\sqrt{3}} \frac{2}{1+\mathrm{u}^2}\,\mathrm{du}$$

$$= [2\arctan(\mathrm{u})]\Big|_1^{\sqrt{3}} = 2\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \frac{\pi}{6}$$